Sediment Transport Capacity of Overland Flow

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This article is reprinted from the TRANSACTIONS of the ASAE (Vol. 28, No. 3, pp. 755-762, 1985)
Published by the American Society of Agricultural Engineers, St. Joseph, Michigan

ABSTRACT

OVERLAND flow runoff can be either laminar or turbulent depending on the Reynolds number. The rate of soil erosion may be limited by the sediment transport capacity which depends on the type of flow. Sediment transport equations based on velocity were found to give different results than those based on shear stress. Most of the sediment transport equations developed for turbulent streams should not be applied to soil erosion by overland flow. A general relationship supported by dimensional analysis was derived. The recommended sediment transport capacity relationship can be written as a power function of slope and discharge and the range of exponents was defined from empirical relationships.

INTRODUCTION

Soil erosion by rainfall is one of the major hazards threatening the productivity of farmlands. The physical processes governing the movement of sediments by rainfall are very complex. The rate of soil erosion depends mainly on the detachment of soil particles and on the transporting capacity of overland runoff. Several sediment transport equations and soil loss relationships have been developed both from experimental studies in laboratories under simulated rainfall (Kilinc, 1972) and from statistical and regression analysis using field data (Wischmeier and Smith, 1978). Recently, some researchers have recommended the use of well-known bed-load equations to predict soil erosion losses from overland flow. For example, Komura (1976) used the Kalinske-Brown relationship and obtained fair agreement with observed data though his data set was relatively limited. The Meyer-Peter and Muller equation has also been suggested by Li (1979) for overland flow. Several sediment transport equations have been examined by Alonso, Neibling and Foster (1981) to determine how well they fit observed data on concave slopes.

Fundamental relationships for sediment transport in turbulent stream flows have been used as a basis for the analysis of sediment transport capacity by overland flow. However, since sheet flows are generally classified as laminar a theoretical analysis is required in order to determine if a sediment transport formula derived for turbulent streamflow is also applicable to laminar sheet flow.

Article was submitted for publication in July, 1984; reviewed and approved for publication by the Soil and Water Div. of ASAE in December, 1984.

The contribution of the first author was supported by the Natural Sciences and Engineering Research Council of Canada.

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Therefore, the purpose of this study was to investigate the applicability of several sediment transport equations under various hydraulic conditions, including laminar sheet flow. In the first part of this paper, the hydraulic characteristics relevant to sediment transport are summarized to clearly point out the differences between turbulent and laminar flows. Then several sediment transport formulas valid for turbulent streamflow are examined under laminar sheet flow conditions. These transformed relationships are compared with regression equations obtained from experimental studies of soil erosion.

OVERLAND FLOW CHARACTERISTICS

In natural fields, overland flow occurs when the rainfall intensity is in excess of the infiltration rate of the soil. A very thin film of water covers the soil surface and it is referred to as the laminar sheet flow. As the runoff rate increases downslope, the flow converges into micro-scale channels called rills which gradually develop until they form large-scale channels called gullies. In a recent study, Thorne (1984) also defined ephemeral gullies in arable fields. These gullies are formed by the concentraiton of surface runoff and are obliterated each year by normal tillage.

Gully, rill and sheet runoff have different hydraulic properties depending on the relative magnitude of inertia and viscous forces. The ratio of these two types of forces defines the Reynolds number Re. When the inertia forces largely overcome the viscous forces, such as flows in rivers and gullies, the Reynolds number is large and the flow is turbulent. In the case of thin overland runoff, the viscous forces overcome the inertia forces and the flow is called laminar. In sheet flows, perturbations induced by raindrop impact and surface roughness are greatly attenuated due to the large magnitude of viscous forces at low Reynolds numbers. These forces damp out the velocity fluctuations caused by these disturbances and sheet flows over rough boundaries remain laminar until a critical value of the Reynolds number is exceeded.

Another major difference between stream flow and sheet flow is related to the flow depth. For a given particle size, the transport of sediments by saltation and suspension in overland flow is very limited due to the reduced flow depth. The bed load movement, however, may predominate and the most accurate sediment transport rates might be given by bed-load equations, the validity of this statement being precisely the purpose of this investigation.

The principal variables describing overland flow are shown in Fig. 1. The main geometric variables are slope length L and gradient S. The hydraulic variables are rainfall intensity i, flow depth h, mean velocity $\bar{u}$, unit water discharge q and thickness of the laminar sublayer δ'. The parameter generally associated with the sediment discharge q_s is bed shear stress τ_o; the other properties

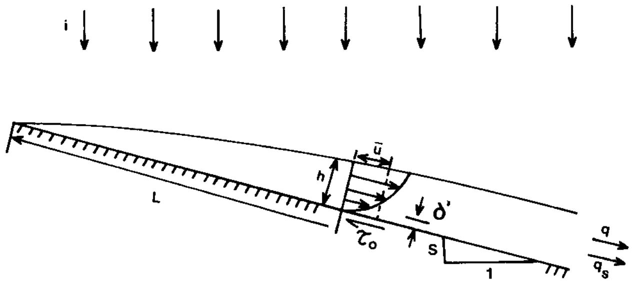

Fig. 1—Overland flow variables.

being gravitational acceleration g, kinematic viscosity ν and specific mass of water ρ.

Fundamental Equations

Two nonlinear partial differential equations derived by de Saint-Venant are generally used to solve the problem of gradually varied unsteady flows. These are the continuity relationship describing the conservation of mass and the momentum equation which is a force equilibrium relationship. In the case of steady overland flow, the continuity equation reduces to:

$$q = \bar{u} h \qquad [1]$$

Considering the principal terms of the momentum equation, the kinematic wave approximation has been recommended by Wooding (1965) and Woolhiser (1975). This approximation states that the slope of the energy line is equal to the soil surface slope, or:

$$S = S_f \qquad [2]$$

in which S_f is the slope of the energy line or friction slope.

The slope of the energy line can be defined by the Darcy-Weisbach relationship:

$$S_f = \frac{f}{8} \frac{\bar{u}^2}{gh} \qquad [3]$$

where f is the Darcy-Weisbach friction factor.

The friction factor f is a function of the Reynolds number and the relative roughness for turbulent flows. The Reynolds number is defined as:

$$Re = \frac{\bar{u} h}{\nu} \qquad [4]$$

As the Reynolds number increases, the flow becomes turbulent and the friction factor is then a function of the roughness of the surface. The roughness of the boundary depends on the sediment size d_s and the thickness of the laminar sublayer δ'. Two important variables are associated with the thickness of the boundary sublayer δ'. These are bed shear stress τ_o and shear velocity U_*. These variables are defined as:

$$\tau_o = \rho g h S_f \qquad [5]$$

$$U_* = \sqrt{\frac{\tau_o}{\rho}} \qquad [6]$$

$$\delta' = \frac{11.6\,\nu}{U_*} \qquad [7]$$

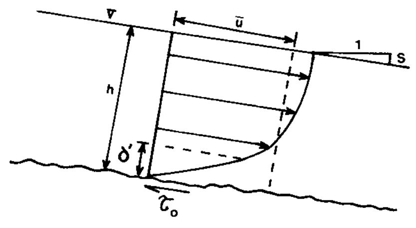

Fig. 2—Turbulent flow over a smooth surface.

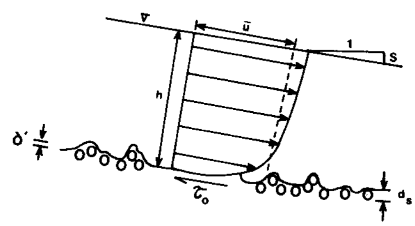

Fig. 3—Turbulent flow over a rough surface.

The ratio of sediment size d_s to the thickness of the laminar sublayer δ' delineates two types of turbulent flow conditions shown in Figs. 2 and 3 for smooth and rough boundaries respectively.

From combining equations [5], [6], and [7], the ratio δ'/d_s is:

$$\frac{\delta'}{d_s} = \frac{11.6\,\nu}{d_s \sqrt{ghS_f}} \qquad [8]$$

This parameter and the Reynolds number are used to define the type of overland flow.

Overland Flow Types

The four principal types of flow relevant to this investigation on soil erosion by overland flow are: (a) laminar sheet flow; (b) turbulent smooth flow governed by the Blasius equation; (c) turbulent rough flow (Manning equation); and (d) turbulent rough flow with very small relative roughness (Chézy equation). This last flow type is not very likely to occur in overland flow. However, it has been included in this analysis since it represents a limiting case for which the Darcy-Weisbach friction factor remains constant. For steady flow conditions, the principal variables related to soil erosion ($\bar{u}$, h and τ_o) are defined as a function of slope and water discharge.

Laminar Flow

In laminar flows with raindrop impact the Darcy-Weisbach friction factor f is related to: (a) the Reynolds number Re, (b) the surface friction coefficient k_o without raindrop impact, and (c) two empirical coefficients A and b for raindrop impact. The following relationship is generally used:

$$f = \frac{K}{Re} = \frac{k_o + Ai^b}{Re} \qquad [9]$$

The values of k_o have been tabulated by Woolhiser (1975) for various surface types and the value $k_o = 24$ is representative of the smooth surface condition. Experimental coefficients A and b have been obtained by Izzard (1944), Li (1972), and Fawkes (1972), and these

TABLE 1. RESISTANCE COEFFICIENTS A AND b FOR RAINFALL.

Reference	A†	b
Izzard (1944)	750	1.33
Li (1972)	118	0.4
Fawkes (1972)	393	1.0

†For i in m per hour.

values are indicated in Table 1. The experimental values of the friction coefficient f obtained by Shen and Li (1973) indicate that for a bare smooth surface, the flow is laminar for Re < 900 and the Blasius law is valid for turbulent smooth flows when Re > 2000. Chen's data (1976) show that k_o can be as large as 50,000 for vegetated surfaces and laminar flow conditions were observed for Reynolds numbers as large as 10^5.

At low Reynolds numbers, the friction parameter K for sheet flows is constant, and the Darcy-Weisbach equation is given by:

$$S_f = \left(\frac{K\nu}{\bar{u}h}\right)\frac{\bar{u}^2}{8gh} \quad [10]$$

The variables $\bar{u}$, h, and τ_o for laminar sheet flows derived from equations [1], [2], [5], and [10] are summarized in Table 2 for comparison with similar relationships valid under turbulent conditions.

Turbulent Flow over a Smooth Surface

For bare soil surfaces the flow becomes turbulent when Re > 2000 and the flow is called hydraulically smooth when the thickness of the laminar sublayer given by equation [7] is much in excess of the size of soil particles ($\delta' > 3\ d_s$). In this case, Keulegan (1938) derived an equation similar to the von Karman-Prandtl logarithmic equation. When the Reynolds number is not too large, this equation can be approximated by the Blasius equation for which the Darcy-Weisbach friction factor becomes:

$$f = \frac{0.316}{Re^{0.25}} \quad [11]$$

For turbulent flows over a smooth surface, the variables $\bar{u}$, h and τ_o given in Table 2 are derived from equations [1], [2], [3], [5], and [11]. The exponents of S for these three variables are identical to those obtained for laminar sheet flows.

TABLE 2. SUMMARY OF FLOW CHARACTERISTICS (VELOCITY, DEPTH, AND SHEAR STRESS).

Velocity	$\bar{u} = c\ S^a\ q^d$			
Type of flow	c		a	d
Laminar (K = constant)	$\left(\frac{8g}{K\nu}\right)^{0.333}$		0.333	0.667
Turbulent, smooth boundary	$\left(\frac{8g}{0.316}\right)^{0.333}$	$\nu^{-0.083}$	0.333	0.417
Turbulent (n = constant)	$\left(\frac{1}{n}\right)^{0.6}$		0.3	0.4
Turbulent (f = constant)	$\left(\frac{8g}{f}\right)^{0.333}$		0.333	0.333
Depth	$h = c\ S^a\ q^d$			
Type of flow	c		a	d
Laminar (K = constant)	$\left(\frac{K\nu}{8g}\right)^{0.333}$		-0.333	0.333
Turbulent, smooth boundary	$\left(\frac{0.316}{8g}\right)^{0.333}$	$\nu^{0.083}$	-0.333	0.583
Turbulent (n = constant)	$n^{0.6}$		-0.3	0.6
Turbulent (f = constant)	$\left(\frac{f}{8g}\right)^{0.333}$		-0.333	0.667
Shear stress	$\tau_o = c\ S^a\ q^d$			
Type of flow	c		a	d
Laminar (K = constant)	$\rho g \left(\frac{K\nu}{8g}\right)^{0.333}$		0.667	0.333
Turbulent, smooth boundary	$\rho g \left(\frac{0.316}{8g}\right)^{0.333}$	$\nu^{0.083}$	0.667	0.583
Turbulent (n = constant)	$\rho g\ n^{0.6}$		0.7	0.6
Turbulent (f = constant)	$\rho g \left(\frac{f}{8g}\right)^{0.333}$		0.667	0.667

Turbulent Flow over a Rough Surface

For turbulent flows an increase of the Reynolds number (or water discharge) raises the water level, and decreases the relative roughness and the friction factor. When the thickness of the laminar sublayer is small compared to sediment size ($\delta' < 5\ d_s$), the flow is hydraulically rough and the logarithmic equation given by Keulegan (1938) applies. The friction coefficient is defined as:

$$\sqrt{\frac{8g}{f}} = C = c_1 \log c_2 \frac{h}{d_s} \quad [12]$$

Approximate power relationships such as the Manning equation, however, are frequently used by hydraulic engineers. Since both equations give similar results for open channel flows without bedforms when the relative roughness ranges between 10 and 10000, the Manning equation (SI units) is used in this study. The equivalent Darcy-Weisbach f is then:

$$\sqrt{\frac{8g}{f}} = \frac{h^{1/6}}{n} \quad [13]$$

in which n is the Manning roughness coefficient. The well-known Manning-Strickler relationship gives the proportionality between the median size of the sediment particles (in mm) at the boundary and the Manning roughness coefficient:

$$n = 0.0132\ d_s^{1/6} \quad [14]$$

The combination of equations [1], [2], [3], [5], and [13] gives the relationships for $\bar{u}$, h, and τ_o shown in Table 2 for turbulent rough conditions (n = constant).

When the relative roughness becomes extremely small, the Darcy-Weisbach equation is equivalent to the Chezy equation ($f = 8g/C^2$). Both coefficients f and C are constant and after combining equations [1], [2], [3], and [5], the variables $\bar{u}$, h, and τ_o are written as a function of S and q. The resulting expressions are listed in Table 2. The relationship $\tau_o \propto \bar{u}^2$ is valid for the Chezy equation and the exponents are slightly different from those derived for the Manning relationship. The exponents of q and S of the velocity relationship are identical and equal to 0.333.

Discussion

The results of this analysis of the hydraulic characteristics are summarized in Table 2. The velocity, the flow depth, and the bed shear stress are written in terms of discharge and slope. The exponent of the slope does not vary significantly for different flow conditions ranging from laminar to turbulent. The exponents of the slope for velocity, flow depth, and bed shear stress are respectively 0.33, −0.33, and 0.67. On the other hand, the exponents of the water discharge vary gradually and differ by a factor 2 between the extreme conditions. The exponents of discharge for velocity and shear stress vary in opposite directions. Indeed, for flow conditions changing from laminar to turbulent flows, the exponent of velocity varies gradually from 0.67 to 0.33 while the exponent of shear stress varies from 0.33 to 0.67. This effect is extremely important if we consider the rate of sediment transport.

SEDIMENT TRANSPORT EQUATIONS

The sediment transport capacity of overland flow under rainfall was investigated to obtain a theoretically sound relationship supported by empirical equations. The method of dimensional analysis was applied to the principal variables related to soil erosion.

Variables and Dimensional Analysis

Sheet erosion is the result of the detachment of soil particles by raindrop impact and transport by overland flow. The eroded soil particles are transported downstream by runoff and the unit sediment discharge is a function of the following variables:

$$q_s = f(L, S, i, \bar{u}, h, q, \tau_o, \tau_c, d_s, \rho_s, \rho, \nu, g) \quad [15]$$

in which τ_c is critical shear stress and d_s is size of soil particles, and the other variables are as defined previously. Among these variables, the first two (L, S) describe the geometry and the next five ($i, \bar{u}, h, q, \tau_o$) are flow characteristics including rainfall intensity. The last six ($\tau_c, d_s, \rho_s, \rho, \nu, g$) are associated with soil and water properties and the gravitational acceleration. Shear stress is difficult to measure in the field and is usually computed from other variables. In a river, the variables S, $\bar{u}$, h, and q are used to describe stream flows because the velocity and depth are generally more easily measured than rainfall intensity i and runoff length L. For this reason, Laursen (1956) suggested the reduction of some sediment transport equations to a function of the variables $\bar{u}$ and h. In the case of soil erosion, however, the variables i and L have important physical significance. The slope and unit water discharge may be more easily measured than the velocity and depth. Therefore, the variables S and q may be more convenient than $\bar{u}$ and h as components of a sediment transport relationship for overland flow under steady-state condition; elimination of the variables $\bar{u}$ and h from the Darcy-Weisbach and the continuity equation $q = \bar{u}h$ is possible.

The critical shear stress value τ_c corresponds to the beginning of motion of the sediment particles. Its evaluation remains a complex problem requiring further investigation, but the fundamental relationship defining the incipient motion of sediments indicates that the critical shear stress is a function of the particle size and the specific masses of water and sediment. Therefore, the sediment size can be replaced by the critical shear stress in a sediment transport equation. The specific masses of water and sediment are nearly constant for particle sizes ranging from clays to gravels. In the case of aggregates, equivalent conditions of shear stress can be defined, for example, using an equivalent diameter, while keeping the same specific mass of sediment in the analysis.

Using these relationships and assuming ρ_s constant, equation [15] reduces to:

$$f(q_s, q, i, L, \rho, \nu, \tau_c/\tau_o, S) = 0 \quad [16]$$

The following Π-terms are obtained from dimensional analysis after L, ρ, and ν are selected as repeating variables:

$$f\left(\frac{q_s}{\rho\nu}, \frac{q}{\nu}, \frac{iL}{\nu}, \frac{\tau_c}{\tau_o}, S\right) = 0 \quad [17]$$

The sediment transport term can be written as a function of the product of the other Π-terms in the form:

$$\left(\frac{q_s}{\rho\nu}\right) = \bar{\alpha}\, S^{\beta}\left(\frac{q}{\nu}\right)^{\gamma}\left(\frac{iL}{\nu}\right)^{\delta}\left(1-\frac{\tau_c}{\tau_o}\right)^{\epsilon} \text{ ; for } \tau_o > \tau_c \quad [18]$$

In this equation, $\bar{\alpha}$, β, γ, δ, and ε are experimental coefficients and the sediment equations based on tractive force and stream power concepts are represents by the term $1 - (\tau_c/\tau_o)$.

Under dimensional form, this equation is transformed to:

$$q_s = \alpha\, S^{\beta} q^{\gamma} i^{\delta} \left(1 - \frac{\tau_c}{\tau_o}\right)^{\epsilon} \quad [19]$$

in which,

$$\alpha = \frac{\bar{\alpha}\rho L^{\delta}}{\nu^{\gamma+\delta-1}} \quad [20]$$

The derivation by Julien (1982) gives a general relationship between the sediment discharge and the dominant geometry and flow variables. The first three factors (S, q, i) represent the potential erosion or transport capacity by overland flow, which is reduced by the last factor reflecting the soil resistance to erosion. When τ_c remains small compared to τ_o, the equation for sediment transport capacity is:

$$q_s = \alpha\, S^{\beta} q^{\gamma} i^{\delta} \quad [21]$$

The sediment transport capacity by turbulent flows in deep channels is not a function of the rainfall intensity, and therefore, $\delta = 0$ in this case.

Empirical Equations

Several empirical equations can be transformed to evaluate the coefficients α, β, γ, δ, and ε. Among the equations analyzed those proposed by Musgrave (1957), Li, Shen and Simons (1973), and Kilinc (1972) include tractive force, stream power, velocity, and discharge relationships. When the variables differ from those of equation [19], the relationships in Table 2 are used for the transformation of $\bar{u}$, h, and τ_o for laminar flows, while the Reynolds number is replaced by $Re = \bar{u}h/\nu$. The results summarized in Table 3 suggest that none of the actual equations is complete since some coefficients are still zero. Consequently, for each particular equation, the number of variables is reduced owing to

TABLE 3. TRANSFORMATION OF EMPIRICAL EROSION EQUATIONS FOR LAMINAR FLOW

$$q_s \sim S^{\beta} q^{\gamma} i^{\delta} \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon}$$

Eq. No.	Reference	Equation	$\alpha^{\dagger}$	β	γ	δ	ε
22	Musgrave (1947)	$q_s = \alpha' S^m L^n i^P$	α'	m	n	p-n	0
23	Zingg (1940)	$q_s \sim L^{1.66} S^{1.37}$	--	1.37	1.66	-1.66	--
24	Wischmeier and Smith (1978) (S>5%)	$q_s \sim L^{1.5} (.00076S^2 + .0053S + .0076)$	--	≅1.7	1.5	-1.5	--
25	Meyer and Monke (1965)	$q_s \sim L^{1.9} S_o^{3.5}$	--	3.5	1.9	-1.9	--
26	Young and Mutchler (1969)	$q_s \sim L^{2.24} S^{0.74}$	--	0.74	2.24	-2.24	--
27	Li et al. (1973)	$q_s = \alpha' \int_o^L \tau_o^2 \, dx$	$3\alpha' \frac{\gamma^2}{5}\left(\frac{K\nu}{8g}\right)^{2/3}$	1.33	1.67	-1	0
28	Komura (1983)	$q_s \sim q^{11/8} i^{1/2} S^{1.5}$	--	1.5	1.38	0.5	0
29	Kilinc (1972)	$q_s = e^{2.05}(\tau_o - \tau_c)^{2.78}$	$e^{2.05} \gamma^{2.78} \left(\frac{K\nu}{8g}\right)^{0.93}$	1.86	0.93	0	2.78
30	Kilinc (1972)	$q_s = e^{0.122} ((\tau_o - \tau_c)\bar{u})^{1.67}$	$e^{0.122} \gamma^{1.67}$	1.67	1.67	0	1.67
31	Kilinc (1972)	$q_s = e^{-3.17} \bar{u}^{3.625}$	$e^{-3.17} \left(\frac{8g}{K\nu}\right)^{1.21}$	1.21	2.42	0	0
32	Kilinc (1972)	$q_s = e^{1.24} \bar{u}^{4.67} Re^{-0.878}$	$e^{1.24} \nu^{0.878} \left(\frac{8g}{K\nu}\right)^{1.56}$	1.56	2.24	0	0
33	Kilinc (1972)	$q_s = e^{-11.6} Re^{2.05} S^{1.46}$	$e^{-11.6} \nu^{-2.05}$	1.46	2.05	0	0
34	Kilinc (1972)	$q_s = e^{11.7} q^{2.035} S^{1.66}$	$e^{11.7}$	1.66	2.03	0	0

†Sediment discharge in pounds per ft-s; multiply by 1.64×10^{-3} for q_s in tons/m·s.

these zero values. The most significant parameters pointed out in this analysis are the slope S and the discharge q. The numerical values of the coefficient β vary from 1.2 to 1.9, and γ, varies from 1.4 to 2.4. The variability of these exponents indicates the complexity of the sediment transport processes since only two major parameters are considered in this analysis. These relationships are in most cases particular to a study area or to the laboratory conditions under which the data were collected. Most equations include rill erosion which also influences the rate of soil erosion.

The well-known Universal Soil Loss Equation cannot be transformed directly into the general equation since the slope factor is written in a quadratic form. The equivalent exponent, however, is expected to vary between 1 and 2, and Julien (1982) found an equivalent exponent value near 1.7 while the discharge exponent 1.5 is an approximate value mostly valid when $S > 0.05$. The Kilinc and Richardson equations cannot define the parameters δ and ε since in their experiments, the soil surface was nearly impervious and also the bed tractive force was much in excess of the critical shear stress value. Throughout these transformations, the number of independent parameters remains the same. For example, equations [23], [25], and [26] based on slope and length have only two independent parameters because $\delta = -\gamma$. This remark is also valid for equations having one independent parameter since for equation [29] $\varepsilon = 3\gamma$ and $\beta = 2\gamma$; for equation [30] $\beta = \gamma = \varepsilon$; and for equation [31] $\beta = \gamma/_2$.

Further fundamental research is required to better define the exponents δ and ε. The coefficients of the general equation obtained by dimensional analysis are kept variable for the purpose of this study and equation [19] is recommended for watershed modeling. The prediction from each equation will be possible, provided the proper set of coefficients is selected from Table 3. Fair estimates can be obtained from a regression

TABLE 4. TRANSFORMED SEDIMENT TRANSPORT EQUATIONS FOR OVERLAND FLOW

$$q_s \sim s^{\beta} q^{\gamma} \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon}$$

Eq. No.	Investigator	Equation	Laminar				Turbulent											
							Smooth Boundary				Manning Equation				Chezy Equation			
			β	γ	ε	Index*	β	γ	ε	Index*	β	γ	ε	Index*	β	γ	ε	Index*
36	Du Boys	$q_s \sim \tau_o(\tau_o-\tau_c)$	1.33	0.66	1	1	1.33	1.17	1	1	1.4	1.2	1	1	1.33	1.33	1	1
37	WES	$q_s \sim (\tau_o-\tau_c)^{m=1.5}$	1	0.5	1.5	0	1	0.88	1.5	0	1.05	0.90	1.5	0	1	1	1.5	0
38	Shields	$q_s \sim Sq(\tau_o-\tau_c)$	1.67	1.33	1	1	1.67	1.58	1	2	1.7	1.6	1	2	1.67	1.67	1	2
39	Schoklitsch	$q_s \sim S^{1.5}(q-q_c)$	1.5	1	--	1	1.5	1	--	1	1.5	1	--	1	1.5	1	--	1
40	Kalinske-Brown	$q_s \sim \tau_o^{2.5}$	1.67	0.83	0	1	1.67	1.46	0	2	1.75	1.5	0	2	1.67	1.67	0	2
41	Meyer-Peter et al.	$q_s \sim (\tau_o-\tau_c)^{1.5}$	1	0.5	1.5	0	1	0.88	1.5	0	1.05	0.9	1.5	0	1	1	1.5	0
42	Bagnold	$q_s \sim \tau_o^{0.5}(\tau_o-\tau_c)$	1	0.5	1	0	1	0.88	1	0	1.05	0.9	1	0	1	1	1	0
43	Engelund-Hansen	$q_s \sim \tau_o^{1.5}\bar{u}^2$	1.67	1.83	0	2	1.67	1.71	0	2	1.65	1.7	0	2	1.67	1.67	0	2
44	Inglis-Lacey	$q_s \sim \bar{u}^5 h^{-1}$	2	3	0	0	2	2.5	0	0	1.8	1.4	0	2	2	1	0	0
45	Yalin ($\tau_o \cong \tau_c$)	$q_s \sim \tau_o^{0.5}(\tau_o-\tau_c)^2$	1.67	0.83	2	1	1.67	1.46	2	2	1.75	1.5	2	2	1.67	1.67	2	2
46	Yalin ($\tau_o >> \tau_c$)	$q_s \sim \tau_o^{0.5}(\tau_o-\tau_c)$	1	0.5	1	0	1	0.88	1	0	1.05	0.9	1	0	1	1	1	0
47	Chang et al.	$q_s \sim \tau_o\bar{u}$	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
48	Barekyan	$q_s \sim Sq\bar{u}$	1.33	1.67	0	2	1.33	1.42	0	2	1.3	1.4	0	2	1.33	1.33	0	1
49	Pedroli	$q_s \sim \tau_o^{1.6}h^{0.2}$	1	0.6	0	0	1	1.05	0	0	1.06	1.08	0	0	1	1.2	0	0

*The index represents the number of exponents within the ranges: $1.2 < \beta < 1.9$; and $1.4 < \gamma < 2.4$.

equation such as given by Kilinc (1972). For example, excellent results were obtained by Julien (1982) with the use of the discharge and slope formula (equation 34).

The formation of rills locally increases the unit water discharge q and the resulting erosion rate is expected to be larger than for uniform flow conditions. The rill erosion data collected by Kilinc were analyzed by Julien and Simons (1984) and once the volume of rill erosion was subtracted from the total erosion, the following regression equation was obtained:

$$q_s \sim S^{1.31}\, q^{1.93}\, ;\, (R^2 = 0.96) \ldots\ldots\ldots\ldots [35]$$

Both exponents for S and q are smaller than those for soil erosion including rills (equation [34]). This equation can be used for comparison with sediments transport equations for laminar sheet flow.

Applicability of Sediment Transport Equations for Turbulent Flows

Most of the well-known sediment transport equations originally derived for turbulent stream flows can be transformed to determine their applicability to natural conditions in overland flows.

This analysis included the transformation of the sediment transport equations suggested by Du Boys (1879), O'Brien-Rindlaub (1934), WES (1935), Shields (1936), Schoklitsch (1934), Kalinske-Brown (1949), Meyer-Peter and Müller (1948), Bagnold (1956), Engelund-Hansen (1967), Inglis-Lacey (1968), Yalin (1977), Chang et al. (1967), Barekyan (1962), and Pedroli (1963). The Einstein bedload equation has not been treated separately since it agreed very well with the Yalin and the Meyer-Peter and Müller equations.

The sediment transport capacity was investigated assuming that sediment size, fluid properties and gravitational acceleration were constant. Particular attention was focused at the values of β and γ which are the exponents of the slope and water discharge in equation 19. For each of the four types of flow described previously, the transformed relationships are summarized in Table 4 as follows: (a) laminar sheet flow; (b) turbulent flow over smooth surface as given by the Blasius equation; (c) turbulent flow described by the Manning equation; and (d) turbulent flow with constant Darcy-Weisbach friction factor f or Che'zy coefficient.

The last column for each type of flow represents an index of fitness of these basic equations with the observed value of exponents. This index is equal to the number of parameters (β, γ) enclosed within the ranges of empirical coefficients as determined in the previous section ($1.2 < \beta < 1.9$ and $1.4 < \gamma < 2.4$). The higher the index, the better this equation should compare with observed data. Conversely, when the index is equal to zero, the given equation is expected to be a poor relationship to predict the sediment transport capacity of overland flow.

The results in this table show that the exponents β of most relationships are within the range of observed values while the exponents γ are usually too small to fall within the range of empirical coefficients. These results were anticipated since it was demonstrated in the analysis of flow characteristics summarized in Table 2 that the exponent of slope remains constant while the exponent of discharge varies for different flow conditions.

CONCLUSIONS

This study points at the relationship existing between the mechanics of overland flow and soil erosion by rainfall. The authors address the problem of estimating the sediment transport capacity for rainfall erosion based on the mechanics of overland flow. The results of this study are of particular interest to:

1. soil scientists concerned with the decrease of productivity of agricultural land caused by rainfall erosion;
2. watershed modelers looking for a better sediment transport capacity relationship for mathematical simulation of erosion processes;
3. sedimentologists interested in the mechanics of sediment transport for both laminar and turbulent flows.

A general sediment transport relationship supported by dimensional analysis ($q_s \sim s^{\beta} q^{\gamma} i^{\delta} [1 - \tau_c/\tau_o]^{\epsilon}$) is recommended and the major conclusions of the study are summarized as follows: the analysis of overland flow characteristics encompasses a wide variety of flow conditions varying from laminar sheet flows on uniform soil surfaces to turbulent flows in rills and gullies. When the principal hydraulic variables of flow depth, velocity, and bed shear stress are written in terms of slope and discharge, the exponent of slope remains constant for these various types of flow. For different types of flow the exponents of discharge for the velocity and shear stress were shown to vary in opposite directions. Therefore, for predicting soil erosion the applicability of sediment transport relationships derived for turbulent flows depends on whether the sediment transport capacity is a function of velocity or shear stress.

Most of the sediment transport equations used for turbulent flow in streams should not be applied to rainfall erosion in laminar sheet flows. Among the equations examined, only those proposed by Engelund-Hansen and Barekyan seem relevant for predicting soil erosion losses by overland runoff. The formulas suggested by Shields, Kalinske-Brown and Yalin might also be considered though the exponent of discharge is clearly too small in the case of laminar sheet flow. The other equations generally underestimate the parameters β and γ and are irrelevant to predict rainfall erosion.

The range of values of the exponents of slope β and discharge γ were well-defined in this analysis of empirical relationships. The transformation of several sediment transport equations for overland flow can be summarized as follows: the empirical exponent β varies between 1.2 and 1.9 while the exponent γ ranges from 1.4 to 2.4. The variation of these exponents indicates the complexity of soil erosion processes and the specificity of each relationship for the site-specific conditions under which they were derived. Moreover, these exponents are function of the formation of rills and this study shows that both exponents β and γ of the sediment transport capacity relationship increase when the rills develop.

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LIST OF SYMBOLS

A	coefficient for raindrop impact
a, b, c, d	coefficients
r, r_1, r_2	constants
C	Chézy coefficient
d_s	size of sediment
f	Darcy-Weisbach friction factor
g	gravitational acceleration
h	flow depth
i	rainfall intensity
k_o	surface friction coefficient
K	friction parameter for laminar sheet flow
L	slope length
n	Manning coefficient
q	unit water discharge
q_c	critical unit water discharge
q_s	unit sediment discharge
R^2	coefficient of determination
S	slope
S_f	slope of the energy line
u	velocity at a distance y from the water surface
$\bar{u}$	mean velocity
U_*	shear velocity
$\alpha,\beta,\gamma,\delta,\varepsilon$	coefficients of the sediment transport equation
δ'	thickness of the laminar sublayer
γ	dynamic viscosity
ν	kinematic viscosity
ρ	specific mass of water
ρ_s	specific mass of sediments
τ_o	bed shear stress
τ_c	critical bed shear stress